Lesson 1-3

Formulas
Definition: In the coordinate plane, the horizontal number line (called the x-axis) and the vertical number line (called the y-axis) interest at their zero points called the Origin.
The Distance Formula

The distance $d$ between any two points with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Example:** Find the distance between (-3, 2) and (4, 1)

$x_1 = -3$, $x_2 = 4$, $y_1 = 2$, $y_2 = 1$

$$d = \sqrt{(-3 - 4)^2 + (2 - 1)^2}$$

$$d = \sqrt{(-7)^2 + (1)^2} = \sqrt{49 + 1}$$

$$d = \sqrt{50} \text{ or } 5\sqrt{2} \text{ or } 7.07$$
Midpoint Formula

In the coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates \((x_1, y_1)\) and \((x_2, y_2)\) are \(\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\).

Example: Find the midpoint between \((-2, 5)\) and \((6, 4)\)

\(x_1 = -2, \quad x_2 = 6, \quad y_1 = 5, \quad \text{and} \quad y_2 = 4\)

\[M = \left(\frac{-2 + 6}{2}, \frac{5 + 4}{2}\right)\]

\[M = \left(\frac{4}{2}, \frac{9}{2}\right) = \left(2, \frac{9}{2}\right)\]
Slope Formula

**Definition:** In a coordinate plane, the slope of a line is the ratio of its vertical rise over its horizontal run. \( \frac{\text{rise}}{\text{run}} \)

**Formula:** The slope \( m \) of a line containing two points with coordinates \((x_1, y_1)\) and \((x_2, y_2)\) is given by the formula \( m = \frac{y_2 - y_1}{x_2 - x_1} \) where \( x_1 \neq x_2 \).

**Example:** Find the slope between (-2, -1) and (4, 5).

\[
x_1 = -2, \quad x_2 = 4, \quad y_1 = -1, \quad y_2 = 5
\]

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{4 - (-2)} = \frac{6}{6} = 1
\]
Describing Lines

- Lines that have a positive slope rise from left to right.
- Lines that have a negative slope fall from left to right.
- Lines that have no slope (the slope is undefined) are vertical.
- Lines that have a slope equal to zero are horizontal.
Some More Examples

Find the slope between (4, -5) and (3, -5) and describe it.

\[ m = \frac{-5 - (-5)}{4 - 3} = \frac{0}{1} = 0 \]

Since the slope is zero, the line must be horizontal.

Find the slope between (3,4) and (3,-2) and describe the line.

\[ m = \frac{4 - (-2)}{3 - 3} = \frac{6}{0} = \emptyset \]

Since the slope is undefined, the line must be vertical.
Example 3: Find the slope of the line through the given points and describe the line.

(7, 6) and (–4, 6)

**Solution:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 6}{(-4) - 7} = \frac{0}{-11} = 0
\]

*This line is horizontal.*
Example 4: Find the slope of the line through the given points and describe the line.

\((-3, -2)\) and \((-3, 8)\)

Solution:

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-2)}{(-3) - (-3)} = \frac{10}{0} \text{ undefined}
\]

This line is vertical.
Practice

- Find the distance between (3, 2) and (-1, 6).
- Find the midpoint between (7, -2) and (-4, 8).
- Find the slope between (-3, -1) and (5, 8) and describe the line.
- Find the slope between (4, 7) and (-4, 5) and describe the line.
- Find the slope between (6, 5) and (-3, 5) and describe the line.