Roots and Zeros

7.5

page 371

Look at the Concept Summary on Zeros, Factors and Roots

Look at the Fundamental Theorem of Algebra,

and its Corollary on page 372
Solve \( x^2 + 2x - 48 = 0 \). State the number and type of roots.

\[
x^2 + 2x - 48 = 0
\]

\[
(x + 8)(x - 6) = 0
\]

\[
x = -8 \quad x = 6
\]

2 real roots
Solve $3a^3 + 18a = 0$. State the number and type of roots.

$3a^3 + 18a = 0$

$3a(a^2 + 6) = 0$

$3a = 0$  $a^2 + 6 = 0$

$a = 0$  $a^2 = -6$

1 real  $a = \pm \sqrt[4]{-6}$

2 imaginary
Solve \( y^4 - 16 = 0 \). State the number and type of roots.

\[
\begin{align*}
  y^4 - 16 &= 0 \\
  (y^2 - 4)(y^2 + 4) &= 0 \\
  (y - 2)(y + 2)(y^2 + 4) &= 0 \\
  y = &-2 \quad y = 2 \\
  y^2 + 4 &= 0 \\
  y^2 &= -4 \\
  y &= \pm 2i \\
  \text{2 \, real} &\quad \text{2 \, imag.}
\end{align*}
\]
Solve

\[ a^4 - 81 = 0 \]

\[(a^2 - 9)(a^2 + 9) = 0\]
\[(a-3)(a+3)(a^2 + 9) = 0\]

\[ a = 3 \quad a = -3 \quad a^2 = -9 \]

2 real \quad a = \pm 3i \quad 2 \text{ imag}
7.4 Remainder & Factor Theorem.ppt

slides 17, 18 follow
Finding the zeros of a polynomial function

- \( f(x) = x^3 - 2x^2 - 9x + 18 \).
- One zero of \( f(x) \) is \( x = 2 \).
- Find the others!
- Use synthetic div. to reduce the degree of the polynomial function and factor completely.
- \( (x-2)(x^2-9) = (x-2)(x+3)(x-3) \)
- Therefore, the zeros are \( x = 2, 3, -3 \)!!!
Your turn!

- $f(x) = x^3 + 6x^2 + 3x - 10$
- $X=-5$ is one zero, find the others!

- The zeros are $x=2,-1,-5$
- Because the factors are $(x-2)(x+1)(x+5)$
Find all of the zeros of \( f(x) = x^3 - x^2 + 2x + 4 \).

\[
f(x) = x^3 - x^2 + 2x + 4
\]

\[
\begin{array}{c|cccc}
& -1 & 1 & -1 & 2 & 4 \\
-1 & & 1 & -1 & 2 & 4 \\
\hline 
& 1 & -2 & 4 & 0 \\
\end{array}
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}
\]

\[
= \frac{2 \pm \sqrt{-12}}{2}
\]

\[
= \frac{2 \pm 2i\sqrt{3}}{2}
\]

\[
= 1 \pm i\sqrt{3}
\]

Zeros are \(-1, 1 \pm i\sqrt{3}\).
**Answer:** Thus, this function has one real zero at $-1$ and two imaginary zeros at $1 + i\sqrt{3}$ and $1 - i\sqrt{3}$. The graph of the function verifies that there is only one real zero.

\[ f(x) = x^3 - x^2 + 2x + 4 \]
Find all of the zeros of $f(x) = x^3 - 3x^2 - 2x + 4$.

\[ f(x) = x^3 - 3x^2 - 2x + 4 \]

From the graph, $1$ is a zero.

\[ \begin{array}{cccc}
1 & -3 & -2 & 4 \\
1 & -2 & -4 & \\
1 & -2 & -4 & 0 \\
\end{array} \]

\[ x = 2 \pm \sqrt{(-2)^2 - 4(1)(-4)} \]
\[ = 2 \pm \sqrt{20} \]
\[ = 2 \pm 2\sqrt{5} \]

Zeros are $1, 1 \pm \sqrt{5}$.
Answer: 1, 1 - \sqrt{5}, 1 + \sqrt{5}
Write a polynomial function of least degree with integral coefficients whose zeros are -2, 1, 3

\[ f(x) = (x+2)(x-1)(x-3) \]
\[ = (x^2 + x - 2)(x - 3) \]
\[ = x^3 + x^2 - 2x - 3x^2 - 3x + 6 \]
\[ = x^3 - 2x^2 - 5x + 6 \]